ACCEPTED MANUSCRIPT

Impact of slope on dynamics of running and climbing

To cite this article before publication: Jason Brown et al 2019 Bioinspir. Biomim. in press https://doi.org/10.1088/1748-3190/ab4467

Manuscript version: Accepted Manuscript

Accepted Manuscript is "the version of the article accepted for publication including all changes made as a result of the peer review process, and which may also include the addition to the article by IOP Publishing of a header, an article ID, a cover sheet and/or an 'Accepted Manuscript' watermark, but excluding any other editing, typesetting or other changes made by IOP Publishing and/or its licensors"

This Accepted Manuscript is © 2019 IOP Publishing Ltd.

During the embargo period (the 12 month period from the publication of the Version of Record of this article), the Accepted Manuscript is fully protected by copyright and cannot be reused or reposted elsewhere. As the Version of Record of this article is going to be / has been published on a subscription basis, this Accepted Manuscript is available for reuse under a CC BY-NC-ND 3.0 licence after the 12 month embargo period.

After the embargo period, everyone is permitted to use copy and redistribute this article for non-commercial purposes only, provided that they adhere to all the terms of the licence <u>https://creativecommons.org/licences/by-nc-nd/3.0</u>

Although reasonable endeavours have been taken to obtain all necessary permissions from third parties to include their copyrighted content within this article, their full citation and copyright line may not be present in this Accepted Manuscript version. Before using any content from this article, please refer to the Version of Record on IOPscience once published for full citation and copyright details, as permissions will likely be required. All third party content is fully copyright protected, unless specifically stated otherwise in the figure caption in the Version of Record.

View the article online for updates and enhancements.

Jason M. Brown, Delvin Peterson, John Schmitt, Nick Gravish and Jonathan E. Clark

Abstract. By combining biological studies and modeling work, the dynamics of running on horizontal terrain and climbing pure vertical surfaces have been distilled down to simple reduced order models. These models have inspired distinct control and design considerations for robots operating in each terrain. However, while the extremes are understood, the intermediate regions of moderate slopes have yet to be fully explored. In this paper, we examine how cockroaches vary their behavior as slope is changed from horizontal to vertical, with special care to examine individual leg forces when possible. The results are then compared with a Lateral Leg Spring based (LLS, horizontal running) and Full-Goldman based (FG, vertical running) models. Overall, from the experimental data, there appears to be a continuous shift in the dynamics as slope varies, which is confirmed by similar behaviors exhibited by the LLS and FG models. Finally, by examining the stability and efficiency of the models, it is shown that there are stability limits related to the amount of energy added by the front versus rear legs. This corresponds to the shift in leg usage demonstrated by the biological experiments and may have significant implications for the design and control of multi-modal robotic systems.

1. Introduction

In order to synthetically capture the fast, flexible, and fluid locomotion demonstrated by animals, the commonalities of their center of mass motion and ground reaction force profiles have been studied and distilled down into analytically tractable models. The Spring Loaded Inverted Pendulum (SLIP) model has been shown to capture the sagittal plane dynamics of level ground running [1] and has been used extensively for modeling dynamic animal and robot locomotion [2–4]. While impactful in modeling running, a comparable single leg model of sagittal plane climbing has yet to be instantiated by biological study. Unlike the SLIP dynamics, which rely on gravity based parabolic flight motion which disappears at pure vertical motion, biological motion in the lateral plane for both horizontal running and vertical climbing has been shown to be instantiated with the Lateral Leg Spring (LLS) model and the Full-Goldman (FG) model, respectively [5,6]. While these models were developed through distinct biological studies, they share many similarities, as both are defined as bipedal models with springy legs where lateral/pendular motions appear critical to stability. While similar, these models have yet to be compared as they have been restricted to the extremes of locomotion (horizontal and vertical).

Analyses of spring-mass based models of cockroach locomotion in the transverse plane have shown both the LLS and FG models to be inherently stable on level ground [7], on inclines [8], and on pure vertical surfaces [9, 10]. It seems that the sprawled posture leg deployment of the cockroach legs, in combination with the natural springiness of the musculo-skeletal system are important for stabilizing cockroach locomotion. In fact, studies of cockroach response to a large lateral impulse show that they began to generate restorative forces faster than would be possible using neural signals [23]. This indicates that the leg morphology and compliance of the cockroach itself naturally generates these forces to stably restore the insect to its natural gait.

These reduced order models have inspired the design and control of numerous robotic platforms. The SLIP model inspired platforms able to approach animal-like velocities, including: Raibert's hoppers, which were first platforms to achieve stable running locomotion with Froude Numbers greater than 1 (which formally differentiates fast walking from running) [11], iSprawl able to move at 15 body lengths per second $(\frac{BL}{s})$ [12], and MIT's Cheetah II (with a mass of 33 kg) able to move at 9.1 $\frac{BL}{s}$ (6.4 $\frac{m}{s}$) [13]. Additionally, the Full-Goldman model has inspired the fastest legged climbing robots which have reached velocities of 60cm/s [10] and 1.95 $\frac{BL}{s}$ [14].

While the whole body dynamics can be distilled down to point mass monopedal or bipedal models, real multi-legged systems have significant flexibility in how they produce these dynamics. Different gaits, such as trotting, cantering, and galloping, while distinct from a controls prospective, are all captured by reduced order models [15]. However, the power consumption, stability, and other dynamic properties between these gaits can vary. A first step towards integrating these models was the extension of the SLIP model from a monopedal model to a compliant bipedal model, where Geyer et al. demonstrated that a single model could capture multiple gaits [16]. Many biological systems utilize 4 or 6 legs,

which provides even more flexibility in leg use, but currently there is limited understanding of potential benefits of leveraging this flexibility.

These concise mathematical models and the robots they have inspired are typically limited to study within a single domain, i.e. either horizontal ground or vertical climbing locomotion. In contrast to this, animals are able to traverse through discrete or continuous terrain changes, switch between multiple domains (walking, running, hopping, climbing), and handle the intermediate transition regions between these shifts. A few synthetic platforms have been able to capture some aspects of this multi-modality, with specific examples including RiSE [17] which was able to walk and climb, SCARAB [18], which was able to run and climb at over 1 BL/S, and BOBCAT [19] which at 5kg was the largest platform to be able to both run and climb (although climbing required constraining the out of plane dynamics with a strap). None of these platforms, however, have demonstrated the ability to negotiate the intermediate region between horizontal and vertical.

While the study of flat ground running has a rich history of biological study, modeling, and robotic development, the study of locomotion dynamics on inclines is still relatively nascent. The studies within each domain have shown that the direction of the leg forces change from pushing away from the body for flat ground running to pulling in toward the body for vertical running, which produce major differences between the lateral plane LLS and FG models. However, there is no current data detailing how the leg forces of the animals that have inspired these models change with respect to inclines. In the horizontal domain, animals like the cockroach use their individual leg pairs differently, but it is unclear how the leg forces within individual leg pairs transition from pushing to pulling (discrete or continuous transition) as the incline changes, and what implications these transitions may have for the associated reduced order models. While past modeling work extended the LLS model to study pushing or pulling based locomotion on slopes [8] and previous experimental work with a FG-based climbing robot examined the running performance as slope was decreased from vertical to horizontal [14], the fundamental LLS or FG models must be extended to directly examine the dynamics as slope is changed. Therefore, new models are required to explore the mechanisms by which the dynamics change, whether it be from changes in gait, frequency, or individual leg function.

In this paper, we examine the locomotion of cockroaches on the intermediate region between flat ground and vertical (i.e slopes) to determine if there are discrete or continuous changes in gaits (i.e. control strategy), with insights then validated with simulation. The remainder of the paper is organized in the following manner. Section 2 describes the experimental setup and results from cockroach experiments running on slopes from 0 to 90°. Section 3 outlines the foundation for the new dynamic models introduced in the paper, a hexapedal model based upon the LLS model and a quadrupedal Full-Goldman inspired model, which will enable exploration into the role of individual leg usage as a function of slope as well as flexibility in power addition. Section 4 presents the results for the hexapedal model followed by Section 5 which discusses the Full-Goldman modeling. A discussion of the overall implications from this work is given in Section 6 with the conclusions presented in Section 7.

2. Cockroach Slope Experiments

Legged locomotion of insects such as cockroaches requires coordination of limb kinematics and generation of appropriate forces when in contact with the ground. Previous studies have identified that running on the level typically involves generating limb forces that are pointed laterally away (distally) from the body [5]. However, when climbing a vertical surface the leg cycles remain the same, but the fore- and mid-limb forces are directed inward (proximally) to maintain appropriate normal and shear forces between the foot and surface [6]. Thus, somewhere between the extremes of level ground (0°) and vertical surfaces (90°) the direction of limb contact forces changes at least for the fore- and mid-limbs. We aim to determine how force production varies between the two extremes and whether there is a smooth transition between pushing to pulling contact forces. Towards this goal, in this section, we describe biological climbing experiments with cockroaches to determine how individual leg forces change as the climbing substrate is varied between level and vertical.

2.1. Methods and Materials

We maintained a colony of a tropical cockroach (*Blaberus discoidalis*) in ambient conditions with ready access to food and water. Animals were kept on a 12-hour on, 12-hour off light cycle and were handled minimally. We performed climbing experiments with 4 individuals (mass 2.40 ± 0.04 g).

Locomotion experiments were performed on a custom trackway, shown in Fig. 1, which included a three-axis force platform. The trackway was mounted on a pivoting base and could be fixed at angles ranging from 0° to 90° . A high-speed camera (AOS technologies) was mounted above the force platform and fixed to the trackway such that it rotated with the assembly. A 45° mirror was mounted next to the trackway and in the view of the camera to provide two camera views of the animal as it crossed the platform.

The interior dimensions of the trackway measured 27 cm \times 6.5 cm and one end contained an enclosed room to which the cockroaches were lured. The substrate was a plastic material with holes every 1 mm which enables cockroaches to engage claws and use their adhesive pads for climbing. In the middle of the trackway, and 10 cm from the starting location, was a custom built three-axis force sensor which measured force normal to the plate, as well as in the lateral and fore-aft direction of the trackway.

The force plate was constructed from four brass beams of rectangular cross section as per the instructions in [20]. Slots were cut out from the beam walls in locations where strain gauges would be located to concentrate strain. Eight strain gauges were arranged around the cutout slots for each of the three axes of measurement. The strain gauges of each axes were configured in a full-bridge configuration and were measured using the INA125A instrumentation amplifier chip with a gain value of 10,000. The output of each of the INA125A amplifiers were directed to a USB data acquisition device (National Instruments, USB-6008) sampling all three axes at 10kHz. A custom LabView program coordinated force and video measurement.



Figure 1: Top). Testing Apparatus including 6 axis force plate, a pivoting base allowing for the setup to be oriented at any angle between horizontal and vertical, a high speed camera along with a 45° mirror to capture both top and side view of cockroach, and an enclosed room for cockroach to run to. Bottom). Cockroach running along trackway with plastic surface (with 1.0 mm spaced holes) highlighted as well as demonstrating the application of the 45° mirror.

Data analysis was performed in Matlab through a custom script. The touchdown and touchoff locations for the fore and rear legs were manually identified and the touchdown force was estimated by taking the mean of the force readings across an 8 ms time interval while the single leg interacted with the force plate. This time interval was chosen to provide electrical noise rejection without removing touchdown force dynamics. Individual leg forces for the fore and rear legs were taken from the first step onto and the last step off the force plate (respectively). Using this experimental setup, it is not possible to directly measure the individual middle leg forces. Overall forces were captured when the animal was fully on the force platform and we measured the mean and standard deviation of the forces. Measurements were taken at track orientations of 0° (horizontal), 15° , 30° , 45° , 60° , and 90° (vertical),



Figure 2: The mean forces in the normal, fore-aft and lateral directions with the theoretical total body force in each direction shown in black, the best fit lines for the experimentally determined front and rear forces shown in light and dark blue respectively (with the mean data points at each angle tested shown along with the 95% confidence interval), and the approximated middle forces shown in blue. The mean forces (Normal shown in top row, fore-aft shown in the middle row, and lateral shown in the bottom row) as the slope is varied. The definition of positive force direction is shown for each leg in each direction with the right figures in each row, with the theoretical total force shown as the black line on each plot.

2.2.1. *Mean Force Component Results* The mean forces in the normal, fore-aft, and lateral directions for the front and rear legs, shown in Fig. 2, are determined from the force measurements described in Sec. 2.1, with the 95% confidence interval shown as the shaded

Impact of Slope on Dynamics of Running and Climbing

regions of Fig. 2. From these measurements, the middle leg force was estimated by taking the difference between the total force and the sum of the front and rear forces. While these measurements were not taken concurrently, this estimate should provide a reasonable approximation of the mean force for the middle legs.

Estimating the total mean forces in each of these component directions, shown as the black lines in Fig. 2, results in three distinct relations. In Fig. 2a, a cosine function captures the relation between normal force and incline, with the mean normal force over a stride being equal to body weight at horizontal and zero normal force at vertical. Fig. 2b shows that the mean fore-aft force should have a sinusoidal relation with incline, with a mean fore-aft force of zero at horizontal and full body weight at vertical. Finally, while there will be some deviation in a given run (with cockroaches not running perfectly straight up the surface), the mean lateral force, shown in Fig. 2c, should average to zero regardless of slope.

The front legs' individual force components, shown by the light blue lines in Fig. 2, are not proportional to the total force components, but demonstrate linear relations with incline for all three components (with R^2 values of 0.9930, 0.9914, 0.9942 for the normal, fore aft, and lateral directions, respectively). The slope magnitudes of the best fit lines appear to be similar, approximately 0.45 body weight per radian $(\frac{bw}{rad})$, while the y intercepts are distinct for each component. At horizontal, the normal force is 0.36 *bw*, the fore aft force is -0.04 *bw*, and the lateral force is 0.20 *bw*. The change from positive to negative represents a change from pushing to pulling in that direction, occurring at the x-intercept of the lines. The incline when this occurs changes based on the component, switching from negative to positive at approximately 50 deg for the normal force, and switching from positive to negative at approximately 30 deg for the lateral component.

The components of the forces from the rear legs, shown as the dark blues line in Fig. 2, are again not proportional to the total force but demonstrate an approximately parabolic relation with the incline angle. The parabolic function best fit line, with the equations shown in each figure, have R^2 values of 0.6342, 0.8663, 0.4847 for the normal, fore-aft, and lateral directions, respectively. All of the forces are positive, with the lowest values occurring at pure horizontal and vertical. For most of the region, the magnitude of the rear force in any direction is the highest of any leg, with the exceptions at low angles where the front legs dominate the normal and lateral forces (with the middle legs also dominating the lateral components) and high angles where the front and middle legs dominate the fore-aft and lateral forces. The y-intercept does vary based on component, with the normal forces having a nominal value of 0.36 bw, the fore-aft nominal value is 0.17 bw, and the lateral component nominal value is 0.05 bw.

The estimated middle leg force components, shown in Fig. 2 with the moderately dark blue line, do not demonstrate any consistent relation with incline as had been seen by the front and rear legs. The normal forces are generally the lowest of the three legs. The fore-aft forces remain slightly negative but near zero for most slopes until increasing in magnitude around 50 deg. The lateral force component appears to increase exponentially with slope, increasing from - 0.25 bw at horizontal to 0.4 bw at vertical with the transition from negative to positive



Figure 3: The overall forces for each leg allowing for comparison of work done by each leg. At steady state, the mean force of all the legs should sum to body weight. The rear legs, which have the largest muscles in the cockroach, generally have the highest force magnitude among the legs. However, the front legs produce the highest forces as the slope approaches vertical, even though more energy should be required to overcome the gain in potential energy.

around 60 deg.

The total forces from each leg, shown in Fig. 3, reflect much of the same results as the component forces in terms of relative magnitudes. Generally, the rear legs have the highest magnitude, with exceptions at low angles (< 10 deg) and high inclines (> 70 deg) where the front legs dominate. The middle leg almost always has the lowest force magnitude.

2.3. Discussion

By examining the sign of forces with respect to each leg, the regions where legs are pushing versus pulling can be determined. As an example of this, based on the definition for foreaft force for the front leg as shown in Fig. 2, it can be seen that a negative fore-aft force corresponds to the leg generating a pushing force (i.e. pointing away from the body) against the direction of travel whereas a positive force corresponds to pulling along the leg. By using this type of analysis, it can be seen that the rear legs only push, thus only providing positive work towards locomotion. The front legs have a unique transition point for each component (normal, fore-aft, and lateral). In the fore-aft direction, the front legs push and thus provide braking near horizontal, then pull and providing positive work for inclines above 10 deg.

Cockroaches generally have ground reaction forces that act through the hip [21] (corresponding to prismatic actuation) which, when combined with their splayed posture, produces significant lateral force generation. By using the lateral force transition point, we

Impact of Slope on Dynamics of Running and Climbing

can see the front legs push against the surface from 0-30 deg then pull for inclines from 30-90 deg and the middle legs push from about 0-60 deg then pull from 60-90 deg.

With the transition points for each leg identified, the overall gait strategy employed at each incline can be estimated. Gaits with all the legs pushing appear to be selected for low slopes (i.e. near horizontal). At moderate slopes, the front legs pull and the middle and rear legs push. Finally, near vertical inclines, the animals pull with the front and middle legs while pushing with their rear legs. The data suggests cockroaches are changing their gait and actuation strategy based on slope, with the change achieved relatively continuously based on the smoothness of the ground reaction force relation to incline.

By considering the total forces, shown in Fig. 3, the relative effort from each leg can be compared. For most inclines, the rear legs output the highest force magnitude while consistently being directed behind the body. Therefore, since they only push, the rear legs produce the most positive work, which matches the muscular structure where the strongest muscles on the cockroach are located within the rear legs [22]. However, the rear legs do not provide the most work as the incline approaches vertical, which should correspond to the region where the most work output is required to overcome the increase in gravitational potential or near horizontal where, based on the gait utilized, the rear legs are the only legs to provide positive work in the fore-aft direction. Near vertical, the rear legs' output actually decreases to the lowest among the three legs, suggesting there must be reasons outside of energy addition for changing the actuation.

Overall, the cockroach appears to be using three different styles of force generation: using a gait with all the legs pushing away from the body near horizontal, a gait with the middle and rear pushing while the front leg pulls for moderate slopes, and a gait with the rear leg pushing while the front and middle leg pull at inclines near vertical. Beyond these gait manifestations, the force magnitude shifts significantly and non- intuitively, with the output from the rear legs (which have the strongest actuators) reducing below the front legs near vertical when the most power addition should be required to overcome the increase in system gravitational potential.

3. Modeling

Based on these results, two important questions were selected for further exploration via modeling. First, what motivates the 3 gait behaviors from horizontal to vertical and second, why would the force from the rear legs reduce significantly at vertical? To explore the 3 distinct gait manifestations of the hexapedal cockroach, multi-legged models are required. Therefore, we extend the LLS model from a bipedal model to a hexapod with each leg able to independently push or pull, with the details and results presented in Sec. 4. To examine the force ratios between the front and rear legs, we extend the FG model to a quadruped which allows the force ratio to be explicitly set, presented in Sec. 5. In this section, we will describe the common elements between the simulations.



Figure 4: a). LLS-Based Tripod Model which uses 2 alternating sets of 3 legs to capture the locomotion exhibited by cockroaches. The legs touch down at fixed angles relative to the body and maintain attachment until the force within the leg reaches zero, which enables between 0-6 legs to be engaged with the surface concurrently. The legs are free to rotate about both the attachment point and COM and are independently capable of pushing away from the body or pulling in towards the body b). Pendular Dynamic Climbing model, the reduced order version of the FG model, which maintains a fixed angular offset between the 2 legs throughout motion, consists of 2 legs using directional adhesion as visualized in c). The legs of the pendular dynamic model are only allowed to pull in towards the body. d). If the model is inverted, while maintaining the fixed angular offset, the actuation can achieve forward motion with pushing. This requires the directional adhesion constraints to be inverted as well, as visualized in e).

3.1. Relevant Reduced Order Models (LLS and FG)

In this study, we develop both a hexapedal LLS-based model and a quadrupedal FG-based model, which are modified to capture the dynamics of locomotion on slopes as well as explore the flexible use of multiple legs to instantiate the reduced order dynamics. Because of their similarities, the common elements between the models will be described here, with differences explained prior to presenting associated results. For both of these models, the body is treated as a point mass, while the mass of all legs is neglected since it amounts to only 13% of the total mass [23]. The legs are represented as springs with variable rest length attached to the center of mass (avoiding the need to consider rotational dynamics).

3.1.1. Leg Attachment and Detachment Conditions The models of this work, unlike past LLS models and only recent Full Goldman based models, utilize attachment and detachment

conditions based on state-based information rather than fixed times. This removes the 50% duty cycle constraint which has simplified many bipedal models into monopeds [8, 10, 14]. For the LLS-based model, the stride of each leg is characterized by a stance phase when the leg contacts the ground and a swing phase when the leg swings to its next location. The stance phase starts based on a fixed clock cycle, but lift-off occurs when each legs' force returns to zero. This fixes the time for the stride phase, but does not fix the length of the stance phase. This allows for duty cycles, or percentage of time a leg contacts the ground, to approximate the typical duty factors of cockroaches (around 52% to 66% on the level [24] and 50% to 74% on vertical slopes [25]).

The FG-based model uses state-based switching conditions for both attachment and detachment by specifying that attachment is initiated and maintained when the foot is pulled down the surface. This extension encodes the concept of directional adhesion [9, 12] which has been shown to be utilized by geckos and cockroaches [12]. With the state-based switching conditions, anywhere from zero to all the legs can be engaged with the surface at once. For these models, directional adhesion will be employed for attachment and detachment of the pendular dynamic models, while timing-based attachment with state-based detachment is used for the LLS-based model.

3.1.2. Leg Actuation Each leg is modeled as a massless, tangentially rigid, linearly elastic spring with a spring force given by:

$$\vec{F}_{leg} = k(|\vec{r}_i| - l_{eq}) \frac{\vec{r}_i}{|\vec{r}_i|} \tag{1}$$

where k is the spring stiffness of the leg, \vec{r}_i is the leg length vector shown in Figure 1, and l_{eq} represents the equilibrium length of the spring, or the leg length which would produce zero force. All of the springs are actuated to enable energy changes in the system. As in other similar models [6, 8, 9, 26] this is accomplished by changing the force-free length of the spring, l_{eq} . While physical springs typically have a fixed equilibrium length, in robots this has been achieved by attaching a traditional spring to an actuated length leg segment [2, 14]. Variation in the force-free length of each leg is modeled in a purely feed forward manner. A sinusoidal actuation pattern is used to both simplify the analysis as well as to instantiate the forces generated by cockroaches which are generally sinusoidal both on the level [22] and on vertical slopes [6].

$$l_{eq} = l_0 - l_{dev} \sin\left(\frac{\pi}{t_{des}}(t - t_{start}) + \phi\right)$$
(2)

In Eq. 2, l_o represents a baseline length for the leg and is taken from experimental observations. The actual starting length in the model differ since the leg must be placed with zero force regardless of the controlled phase shift, ϕ . The amount of deviation in the leg equilibrium length is limited by l_{dev} , the leg actuation parameter. The desired period of the cycle is defined as t_{des} and the phase shift at the start of actuation is defined as t_{start} . Physical observations of cockroaches on the level show typical variations in the distance to foot placement point of about 10% of the average values [27]. In initial trials, to provide

Impact of Slope on Dynamics of Running and Climbing

enough actuation for climbing a slope, an l_{dev} value of at least 50% of l_0 was required. Too small a value leads to an ineffective leg on slopes, and a too large value could lead to places where the equilibrium length would have to be negative, which is physically impossible.

3.2. Equation of Motion

Within the plane of motion, the only forces that act on the body are the leg forces and a component of the gravitational force. The equations of motion, developed using Newton's laws in the inertial frame, are given by:

$$\ddot{x} = \frac{\sum F_x}{m} \tag{3}$$

where
$$\sum F_x = \mathbb{C}_x \left(\sum_{i=1}^2 \vec{F}_{leg_i} \right)$$
 (4)

$$=\frac{\sum F_y}{m} \tag{5}$$

where
$$\sum F_y = \mathbb{C}_y \left(\sum_{i=1}^2 \vec{F}_{leg_i} \right) - mg\sin(\sigma)$$
 (6)

In Eqs. 4 and 6, \mathbb{C} refers to the inertial frame component of the vector and \vec{F}_{leg_i} are the leg forces from Eq. 1. The mass is represented by *m* and the slope of the incline by σ . Assuming appropriate initial conditions, these differential equations can be integrated using a numerical solver. This is done in Matlab using ode45, a numerical ordinary differential equation solver based on an explicit Runge-Kutta 4-5 formula.

ÿ

3.3. Analysis and Optimization

3.3.1. Fixed Point Search While the motion of the model is continuous, it can be discretized by observing the state of the model at foot placement of subsequent steps. In this manner, the state of the system is sampled as it passes through a Poincaré section represented by the instant when each foot is placed. A Poincaré map is a function that outputs the state of the continuous system for the next time the Poincaré section is reached based on the state of the continuous system at the last crossing of the Poincaré section. A fixed point of the mapping is defined by a set of conditions and foot placement parameters that result in the model returning to the same velocity magnitude and heading direction after each stride. For the LLS based model, the model parameters and conditions used in this study will be presented in Sec. 4 while the Full Goldman based parameters and conditions will be discussed in Sec. 5. The stability of the resulting fixed points will be determined from the eigenvalues of this linearized return map.

3.3.2. Optimization By modulating the leg actuation parameter, larger or smaller amounts of energy can be added to the system with the final value found using a Nelder-Mead simplex algorithm. [28, 29]. Using the search algorithm, optimal fixed points were found for each of

Impact of Slo	ope on Dynar	nics of Runni	ing and Climbing
---------------	--------------	---------------	------------------

	-	
Constraint	Туре	Purpose
Fixed point exists	Hard	Desired gaits must return to their original states
		at each step
Fixed point exists at	Hard	Prevent fixed points right at a barrier of existence
105% of δ_0 (Heading An-		
gle)		
$l_{dev_i} \neq 0.9 l_{0_i}$	Hard	Too much actuation could cause an impossible
		negative equilibrium length
$l_{dev_i} eq 0.2 l_{0_i}$	Hard	Too little actuation renders the leg force insignif-
		icant
$\phi_i ot < -90^\circ$	Hard	Too much negative phase shift can cause the leg
		to switch between pushing and pulling function
$\phi_i eq 60^\circ$	Soft	Too much positive phase shift renders the leg
		force insignificant
$\left tan^{-1} \frac{d}{dt} \left l_{eq} - l_{spr} \right _{tTD} \leq 10^{\circ}$	Soft	Too shallow of a departure between the equilib-
		rium and actual spring lengths after leg touch
		down leads to an insignificant force or a switch
		in leg function
$tan^{-1}\frac{d}{dt}\left l_{eq}-l_{spr}\right _{to} \neq 10^{\circ}$	Soft	Too shallow of an approach between the equilib-
		rium and actual spring lengths before leg lift off
		could lead to a missed lift off if the difference
		never returns to zero

Table 1: Additional constraints implemented during optimization based on stability and energy efficiency. Hard constraints can not be violated by the optimizing routine, while soft constraints can be violated with additional penalties added to the function that is being minimized.

the conditions described in Sec. 4 for a variety of pushing combinations, slopes, and heading angles. Optimal fixed points minimized a cost function comprised primarily of the absolute value of the largest eigenvalue, as this governs gait stability. However, several additional cost function components were included to avoid characteristics of non-desirable fixed points as shown in Table 1. These are categorized as either soft or hard constraints. Costs associated with soft constraints are zero until the constraint is violated and subsequently increase as the value extends beyond the constraint. In this way these constraint violations were discouraged, but not prohibited. On the other hand, the hard constraints included a large punitive cost imposed immediately upon constraint violation in addition to the gradually growing cost. These prevented the search algorithm from continuing to violate these constraints. Hard constraints were imposed in cases where violating the constraint would not produce a valid fixed point.



Figure 5: Illustration of Lateral Leg Spring Model state parameters and a complete stride. A stride is comprised of a right stance phase followed by a left stance phase and finishes at the start of the next right stance phase. The parameters employed to uniquely define the resultant trajectory are the touchdown velocity (v_0) and velocity heading angle (δ_0). For the tripod model, three additional parameters are required, specifically the phasing (ϕ) for the front, middle, and hind legs.

4. Alternating Tripod LLS Model

The fundamental LLS model was extended from a bipedal to hexapedal model by replacing the single effective legs of the original model with two sets of tripods in order to explore the effect of differential leg function, as exhibited by the experimental results presented in Sec. 2.2. As described for the LLS model, all the legs of a tripod touch down at the same time and each leg remains in contact until the its ground reaction force goes to zero. The front, middle and back leg each touch down at a distinct globally defined leg angle (since the body receives no moments and thus does not rotate) as shown in Table 2.

The state parameters for the LLS model are $(v_i^{TD}, \delta_i^{TD})$ at foot placement, where v_i^{TD} represents the COM velocity magnitude at TD and δ_i^{TD} represents the TD velocity heading angle, as shown in Fig. 5, which arise from the initial conditions (v_0, δ_0) . When extending the model from a biped to hexapod, the phase shift parameters of the leg actuation (ϕ) discussed in Eq. 2 are also critical in defining a fixed point. To limit the number of parameters, only symmetrical gaits are considered, thus only a single phase shift parameter for each pair of front, mid, and back legs is required to define a fixed point. Additionally, δ_0 was chosen to be swept to explore the impact on stability and efficiency of lateral motion (which in the unperturbed system should be unnecessary). Fixed points were found using a numerical fixed point solver that varies v_0 and each of three ϕ parameters while leaving all other parameters at fixed values given in Table 2 selected to match observed cockroach values. When identifying optimal fixed points, the solver is also allowed to vary the leg actuation parameters l_{dev} to minimize the prescribed cost function.

The experiments performed in Sec. 2.2 captured both the forces and velocities of cockroaches climbing various slopes. Although not shown for brevity, the results indicate cockroaches' preferred speed varies in a linear fashion as the slope varies from horizontal to

Parameter	Value	
k: Leg Stiffness	1.5N/m	
<i>m</i> : Mass	2.5 <i>g</i> [30]	
σ: Incline Slope	$0^{\circ} ightarrow 90^{\circ}$	
v _{des} Desired Velocity	$0.35m/s \rightarrow 0.20m/s$	
t _{des} : Desired Step Duration	$.0364 s \rightarrow .0598 s [6, 31]$	
δ_0 : Heading Angle	$0.00 \text{ rad} \rightarrow 0.45 \text{ rad}^*$	
$\beta_{0,i}$: Foot Placement Angle	F: $\pm 25^{\circ}$ [27] M: $\mp 55^{\circ}$ [27] B: $\pm 120^{\circ}$ [27]	
l_{0_i} : Nominal leg length	F: 0.032 <i>m</i> [27] M: 0.026 <i>m</i> [27] B: 0.019 <i>m</i> [27]	
<i>l_{devi}</i> : Leg Equilibrium Deviation	F: 0.016 <i>m</i> M: 0.013 <i>m</i> B: 0.0095 <i>m</i>	

Impact of S

Table 2: Selected parameters for reduced order model of cockroach motion based on actual cockroach parameters.

vertical from 0.35 $m/s \rightarrow 0.20 m/s$ respectively, while preferring a heading angle of $\delta_0 =$ 0.15 ± 0.10 rad (with an additional 0.10 rad uncertainty based on the yaw angle variance in their body orientation in relation to the inertial frame). Therefore, the desired velocity (v_{des}) and step duration (t_{des}) were interpolated based on the incline slope (σ) which is varied in increments of 5°. Additionally, initial heading angles (δ_0) varied as shown from 0 rad to 0.45 rad in increments of 0.05 rad for each slope, with the range selected to account for the yaw angle variance not instantiated in the point mass model. Varying this parameter allows us to investigate gaits ranging from small to large amounts of lateral oscillation.

4.1. Optimization Results

With the 3 symmetric sets of legs and with each set being able to push or pull, there are a total of 8 combinations of pushing and pulling with the front, middle, and back legs. However, no stable gaits were found with any gait in which the back legs pulled (which would produce purely negative work). The four remaining gaits (shown on the top row of Fig. 6) are, (A) where all legs push, (B) the front and back push while the middle pulls, (C) the middle and back push while the front pulls, and (D) where only the back pushes while the front and middle pull are explored independently, with the leg role determined by the sign of l_{dev} (positive l_{dev} corresponds to pulling while negative l_{dev} corresponds to pushing).

The resulting regions for each of the gait families are shown in Fig. 6, which shows that gaits where only the back leg pushes (D) result in the widest range of incline slopes



Impact of Slope on Dynamics of Running and Climbing

Figure 6: Depiction of the regions with viable gaits using all successful strategies. All successful gaits required at least the rear legs pushing, with the other successful gaits having additional legs pushing. Only rear legs pushing has the largest region and the most overlap with other gaits (essentially overlapping with the entire middle and rear leg pushing region). For stable behavior at low slopes, the only viable gaits pushed with the front legs as well as the rear legs.

and heading angles among the leg functional families, ranging from 90° down to 10° and for the entire range of initial heading angles. The middle and back leg pushing gaits (C) are similarly prevalent for higher slopes (with nearly identical range of slopes), but could only be found for shallower initial heading angles. With all legs pushing (A) (the strategy found experimentally at zero slope), the fixed points are generally found only for low incline slopes and low heading angles, while the front and back leg pushing gaits (B) were only found for high heading angles and low incline slopes. While there is significant overlap between the various gait regions, with up to three different gaits overlapped, there was no point found with all four leg functional families available.



Figure 7: The top row shows the maximum absolute eigenvalue for each gait within the viable region, with larger (and thus lighter) points representing reduced stability while darker points represented increased stability. The bottom figure shows the gait with the best stability at each point tested. For these gaits, the rear pushing gait dominates most of the region above 30° , while the all push and front rear push split the region near horizontal.

4.2. Comparison of Results

As multiple strategies exist for most combinations of slope and heading angle, an optimization is used to select between the available functional gait families, the results of which is shown in Fig. 7. The gait with the smallest eigenvalue at each heading angle and incline slope is indicated on the plot. The first item to note is the entire region has eigenvalues less than one, which suggests the entire region is stable.

At incline slopes of $\sigma = 20^{\circ}$ and above, nearly all of the selected gaits only use (D) the back leg pushing. The back leg pushing (D) is generally preferred for the entire region where it is viable, with other gaits being selected at the boundaries of the space. At lower incline slopes, the selected gait family depends on heading angle, with the all legs pushing gaits (A) selected for heading angles less than $\delta_0 = 0.2$ rad. Higher heading angles use gaits with only the front and back legs pushing (B). The middle and back legs pushing gaits (C) are rarely

Impact of Slope on Dynamics of Running and Climbing

selected since they generally are viable in the same region as the back legs only pushing gaits, which are more stable.

4.3. Discussion

For the entire range tested, there were stable gaits using at least one of 4 viable gaits, often with multiple gaits available. With 3 of the viable gaits, there are a non-zero number of legs doing negative work. From the experimental data presented in Sec. 2.1, the rear legs consistently have significant actuation (which is the only leg to always do positive work). However as more legs do positive work, other legs begin to dominate the actuation, with the front legs having the highest actuation for the gait with only the rear legs pushing.

5. Pendular Dynamic Modeling

Since the FG model or its reduced order version, the pendular dynamic model, has yet to be modeled on slopes, the fundamental bipedal version was augmented to allow the legs to either push or pull using the switching conditions described in Sec. 3.1.1 to be compared with past LLS modeling. A fundamental difference between the LLS model and pendular dynamic model is that the relative angle between the legs (β) is maintained through stance rather than being free to rotate. This means the body rotates about the fixed contact, which approximates body yaw seen in animal running, rather than fixed body orientation with rotating legs. With this sprawl angle β constraint, in order to successfully push up a surface, the pendular dynamic model must begin inverted, as shown in Fig. 4c, which can be achieved with the same model, just with different initial conditions. The second modeling difference between the LLS model and pendular dynamic model is the use of state based attachment rather than the timing based attachment used for the tripod model.

Because of the added model flexibility of state based attachment, the number of state variables increase from two for the LLS model to four $(L, \dot{L}, \theta, \dot{\theta})$, where *L* represents the leg length at TD, \dot{L} represents the linear velocity of the body along the leg, θ represents the angle of the stance leg from vertical, and $\dot{\theta}$ represents the body velocity in the angular direction. For the pendular dynamic model, the two legs are actuated with a 180° phasing offset, meaning when one leg begins retracting, the opposite leg begins extending. With this phasing between the leg's actuation, only a single additional phasing parameter ϕ from Eq. 2 is required to complete the state description.

5.1. One Dimensional Reductions

Since the system relies on an initial offset and fixed angular displacement between the two legs, the pendular dynamic model can easily be condensed down to a one dimensional model, essentially sliding up a rail to climb. This model reduction produces a model similar to Clark et. al's one dimensional model as well as Brown et. al's one dimensional model [9, 32]. This reduction allows for the stability and energy addition to be explored without the influence

Parameter	Description	Value
g	Gravitational Constant	9.81 ms ⁻²
i	Leg Identifier	[0 or 1]
Leg _i	Flag for Attachment of Leg i	[0 or 1]
L _{nom}	Nominal Length of Leg	0.0281 m
L _{dev}	Leg Deviation	$0.0093 L_{nom} m$
k	Spring Stiffness of Leg Spring	6 Nm ⁻¹
β	Sprawl Angle	20 deg
b	Damping Constant	$0.09{ m Nsm^{-1}}$
т	Body Mass	2.0 g
Ω	Driving Frequency	9 Hz
σ	Incline Angle	0 - 90°

Impac

of the pendular motion fundamental to the Full-Goldman model. This decoupling provides insights into the relative impact of the pendular motion compared with the actuation strategy.

5.2. Push Pull Model

The pendular dynamic model is developed to explicitly examine the influence of varying the amount of pushing versus pulling on locomotion. The tripod model was able to demonstrate that there appears to be advantages to pushing and pulling with front and rear legs, respectively, but was not set up to test if there is a limit to the amount of force the rear legs can provide. To explicitly adjust the ratio of pushing verses pulling, a virtual rear leg was included in the pendular dynamic model, as shown in Fig. 8. The attachment point of the rear leg is determined at the moment of TD at the point exactly opposite of the front foot. In order to avoid kinematically constrained motion, the virtual leg imposes no motion constraints on the model (thus the model can viewed as having a pivot at the COM).

To be functionally equivalent to the bipedal climbers, the pushing force of the rear legs is set to be proportional to the expected pulling force from the front legs (which is subsequently reduced by 1-proportion), which can be defined by:

$$|\vec{F}_{rear}| = \gamma |\vec{F}_{biped}| \quad |\vec{F}_{front}| = (1 - \gamma) |\vec{F}_{biped}| \tag{7}$$

where γ is the push pull ratio. These force magnitudes are then directed along the vectors from the foot to the COM.



Figure 8: FG based Push Pull Model, which allows for the ratio of front leg pulling force verses rear leg pushing force to be explicitly set. The front legs are tracked and maintain attachment based on state based switching conditions. At TD, the rear leg foot hold is defined and maintained until the front leg releases. The force from the rear leg is set to be a fraction of the bipedal template dynamics directed from the rear foothold to the COM.

5.3. Model Parameters

In order to compare results with past work, the parameters chosen for the leg length, spring stiffness, damping, mass, l_{dev} , and frequency of actuation, shown in Table 3, were chosen to match the original Full Goldman model. With this model, the climbing velocity is left free, determined by the stride length, rather than a fixed parameter. With this reduced order model, only the impact of wall angle will be examined, which will allow for comparisons to the original Full-Goldman model as well as the fundamental LLS on slopes. Once compared with past LLS modeling, a second dimension, the push pull ratio γ , will be explored.

5.4. Results

In order to compare with previous LLS modeling on slopes, a purely pushing and purely pulling version of the model were simulated over the range of slopes from horizontal to vertical. In the previous work when using pulling, the LLS model was stable for the entire range of slopes, but when using pushing, the model was only stable from -17° to 30° [8]. The FG based model results, shown in Fig. 9, demonstrate some similarities and differences.

Impact of Slope on Dynamics of Running and Climbing



Figure 9: Stability of FG Model pushing away from the body or pulling in towards the body. Pulling in towards the body in stable for almost all inclination angles, only going unstable below 5° . Pushing away from the body in unstable for all slopes tested with the system near neutral stability near 10° .

5.4.1. Comparing Pushing vs Pulling The pulling model has eigenvalues < 1 for almost the entire range with instability occurring at slopes $<5^{\circ}$, with marginal variability in maximum eigenvalue for almost the entire stable region (> 15°). The pushing model has eigenvalues above 1 for all slopes tested, with the system nearing neutral stability (although remaining unstable) at 10°. The pushing and pulling models were furthered examined using the one dimensional reduction described in Sec. 5.1. The models produce identical behavior at each tested point, producing the same speed, efficiency, and stability. This confirms the expected result that from an energetic perspective, that it would be just as viable to push as pull up a surface.

5.4.2. Combined Pushing and Pulling While it has been demonstrated by both the LLS and FG models that purely pulling provides stability for almost all slopes from horizontal to vertical and purely pushing results in instability for all but the slopes near horizontal, it would be extremely inefficient to purely pull or push. Thus the push pull ratio of the combined push pull model described in Sec. 5.2 was swept from 0 (pure pulling) to 1 (pure pulling) as the slope was varied from horizontal to vertical.

The 2 dimensional sweep, shown in Fig. 10, has the line of neutral stability ($\lambda = 1$) labeled. The stable region spans almost the entire range (from vertical to 5°) from a push pull ratio below 0.2. The stability region then extends to higher push pull ratios (up to around 0.75) for slopes from 5° to 30°. As seen with the pure pulling model, the magnitude of the



Figure 10: Maximum Absolute Eigenvalue of Push Pull Model for all wall inclinations and push pull ratios. The stable region ranges from inclines of 5 to 90° for push pull ratios from 0 - 0.13, with an additional region centered around 20° which extends up to 0.8 push pull ratio. The FG based model appears to break down at low slopes (< 20°) where gravity no longer enables large lateral motion with the pendular model.

eigenvalue has limited variation within stable range (ranging from ($\lambda = 0.75 \rightarrow 0.99$).

5.5. Discussion

With the FG-based model, the energetics of the spring mass damper actuation can be isolated from the biologically instantiated pendular dynamics (and thus lateral motion) [6]). With the one dimensional model, there was not a significant difference between pushing and pulling, implying (as expected) that from an energy addition perspective, pushing is as viable as pulling, thus the amount of pushing versus pulling is likely chosen based on stability constraints rather than actuation limitations.

The stability of purely pushing versus pulling can be captured by the bipedal reduced order model using both a standard pendular model and an inverted model (i.e. pulling produces pendular motion while pushing produces inverted pendulum based motion). With the pendular dynamics (using a fixed offset leg) which use gravity to generate lateral motion, gravity will provide a force to reorient the system to climb opposed to gravity. With the inverted model, gravity produces forces to turn the system around (and run with gravity).

When using combined pushing and pulling, the limit of stable rear leg actuation appeared to settle around 20% from vertical to around 60° as seen from the insert in Fig. 10, with

Impact of Slope on Dynamics of Running and Climbing

this limit is extending to 40% at 30°. With the pendular dynamic model, stable motion was not found for horizontal motion, which is likely because the lateral motion seen from the LLS model is necessary for stable motion is created by gravity with the FG model (which disappears at the pure horizontal).

6. Implications

6.1. Design and Control for Stability

Based on the transition points from pushing to pulling for each leg in the experimental data (which for the sprawled posture of the cockroach is approximated from the lateral forces), there appear to be three modes of limb force generation used by the cockroach: all legs pushing from 0-30 deg, middle and rear pushing from 30-60 deg, and rear pushing from 60-90 deg. Of these, the hexapedal simulation matches these results near horizontal (0-30 deg), where the selected gaits favored all legs pushing, and near vertical (60-90 deg) where the rear pushing gait was selected. The model and experimental results differ from 30-60 deg, where the model chose to pull with the middle legs while the experimental results suggested the middle legs push. Within simulation, the middle and rear pushing gait was viable in that region, but was less stable and efficient than the rear pushing gait. However, the simulation does not consider normal forces, which below 60 deg is still more than half the body weight. Since supporting normal weight above the ground requires pushing off the ground, the possible loss in efficiency and stability by pushing with the middle legs is likely outweighed by the need to push with the leg to keep the body from dragging on the surface. By comparing the modeling and experimental results, it appears that cockroaches choose the most stable gait available which allows it to support its body weight in the normal direction.

The transitions between these gaits appear to be relatively continuous and associated with the changes in gravitational loading as slope changes. The first transition occurs when gravitational loading in the *fore-aft* direction increases above 0.5 bw, where more force is required to overcome the increase in potential energy, and the selected gait switches the front legs from braking to propelling the body in the fore-aft direction. The second gait transition occurs when gravitational loading in the *normal* direction reduces below 0.5 bw, where the requirements to push the body away from the surface decrease significantly and the gait selected switches from the middle pushing to the middle pulling.

The gait selected for horizontal running has the front legs producing significant negative work. This is likely for stability as gaits with negative work are the only viable options at horizontal, which matches past experimental work, which showed that, while inefficient, front leg braking was necessary for stable limit cycles [33].

6.2. Control Implications

When climbing vertical surfaces (when the most energy is required to overcome the increase in gravitational potential energy), cockroaches significantly reduced the contribution from their rear legs, which have the most strength of any of the legs. From the push-pull model,

Impact of Slope on Dynamics of Running and Climbing

there is a stability limit to the amount of pushing at vertical at approximately 20% of the total force, which matches the limit of the cockroach pushing with its rear legs at vertical. This stability limit appears to be related to the pendular dynamics of FG style climbing.

Based on these results, the cockroach mechanical design is sub-optimal for climbing, as the rear legs are far stronger than can be utilized. The cockroaches mass and muscle distribution match other running animals, which have stronger rear limbs than front limbs. However, this distribution should also be sub-optimal for producing the forces at horizontal, where the front legs also dominate. One possible explanation for this would be that, while braking on horizontal terrain, the front legs are operating near a singularity, allowing higher output forces, thus behaving more like a strut than a spring.

These results might help explain some struggles of quadrupedal climbing platforms. A select number of quadrupeds have been able to achieve near biological levels of performance (> 0.4 body lengths per sec $(\frac{BL}{s})$), but have generally struggled with stability issues. SCARAB was able to both run and climb at speeds exceeding $(1 \frac{BL}{s})$ but could only go that fast on slopes up to 75° inclines [18]. More recently, BOBCAT, was able to achieve $16 \frac{cm}{s}$ on a pure vertical surface, but required a strap to constrain the out of plane dynamics [19]. Both of these platforms attempted a push/pull ratio of 50% in order to maximize that power output.

The simulation and experimental results provide some insights for designing and controlling these robots to ascend steep slopes. The simulation suggests that asymmetric actuation (where the back legs push less than the front legs pull) might be critical for stable locomotion. For the scale of the cockroach, this corresponded to only pushing with 20% of the total forces. It is of note that the cockroach appeared to push to this stability limit, suggesting the cockroach is still attempting to maximize the power output of each leg within the limits of stability.

6.3. Comparing LLS and FG on intermediate slopes

As seen from the derivation of the equations of motion, the LLS and FG models share considerable similarities, allowing for the same framework to describe both models. However, the models, which both produce significant oscillations perpendicular to the direction of motion, have previously been constrained to studies of horizontal and vertical motions, respectively. In this study, the models were both run on inclines from horizontal to vertical, which allows for direct comparison.

While the models are distinct and were developed based on distinct sets of biological data, they produced similar results on slopes. Both models found purely pushing was unstable for slopes above 30° , while primarily pulling with the front legs was almost universally stable (as seen from Fig. 7 and 9). With combined actuation, both models limited the amount of actuation from the rear legs for higher slopes, which the FG model found was based on stability rather than energetics.

While there are compelling similarities, most of these are found at higher slopes. The largest difference was the FG model's inability to find a stable running gait at pure horizontal, as well as failing to find stable running gaits for pushing gaits at low slopes, both of which

Impact of Slope on Dynamics of Running and Climbing

the LLS model found. This is potentially due to the LLS model enforcing lateral motion via laterally oriented spring mass dampers, which produce lateral motion regardless of orientation to gravity, while the FG model uses pendular dynamics enforced by gravity to produce lateral motions. As the slope approaches horizontal, the projected gravitational force which produces the lateral motion approaches zero.

The FG model failing to capture horizontal running dynamics is not surprising since the model was derived from experimental data of geckos and cockroaches climbing. However, the LLS model producing similar qualitative behavior as the FG model on steep inclines, which extends beyond the derivation from cockroaches running on horizontal terrain, is more surprising. The primary difference between the models is that the FG uses a fixed angular offset between the legs while the LLS model allows the legs to sweep in the angular direction while in stance.

7. Conclusion

In this paper, we present experimental and simulation study of running on slopes ranging from horizontal to vertical. Biological experiments, capturing the ground reaction forces for the front and rear legs, suggest cockroaches use three gait manifestations which are varied with slope. At inclines near horizontal, all the legs push out away from the body which causes the front and middle legs to produce braking force opposed to the direction of travel. At moderate inclines, the middle and rear legs push while the front legs pull in towards the body, causing the front legs to contribute positive work. Finally at inclines near vertical, only the rear legs push while the front and middle legs pull in towards the body. The planar alternating tripod model found 4 viable gaits, the three noted above and a forth where the front and rear legs push while the middle pulls. Using a stability optimization, the selected gaits match the experimental data for low and high inclines and the transition points between the gaits. At the moderate inclines, the animals used a gait the simulation found to be viable but sub-optimal. The need to support the normal body weight not captured in the model likely accounts for the discrepancy.

In addition to the distinct gait manifestations, the forces generated by the legs varied as the incline changed. Morphologically, the rear legs of the cockroach are the strongest, and for most inclines, the rear legs provide the most force. However, at inclines near vertical, where the amount of work output would be expected to increase to account for the system's gain in potential energy, the rear legs produce less force with the front legs dominating. The quadrupedal model was developed to capture the amount of pushing vs pulling ratio explicitly and determined that for stability reasons near vertical the amount of energy addition from the rear legs should remain below 20% of the total force (which matched the ratio used by cockroaches climbing). This asymmetric force generation might help explain the stability issues which dynamic climbing quadrupeds have so far experienced.

Future work includes examining how maneuverability is achieved by biology especially in the near vertical domain. The two models used (FG and LLS), which both qualitatively captured dynamics in the vertical and near vertical domain, produce different constraints on

Impact of Slope on Dynamics of Running and Climbing

locomotion and maneuverability. In the future, combining these models into a hybrid model which can choose to use either constraint could enable more dynamic maneuverability on vertical surfaces while maintaining the ability to passively orient with gravity. In addition to this, the design and control implication of asymmetric force generation will be examined on quadrupedal robotic platforms. Understanding these design compromises between operating in the horizontal and vertical domains should enable faster, more capable multi-modal robotic platforms.

Acknowledgment

We thank Daniel I. Goldman and Robert J. Full for initial discussion and experimental help. We thank Mateo Garcia and Andrei Savu for data collection assistance and force plate fabrication assistance. This work was supported in part by the National Science Foundation Grant Numbers CMMI-1351524 and CMMI-0826583. Any opinion, findings, and conclusions or recommendations express in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

References

- [1] Giovanni A Cavagna, Norman C Heglund, and C Richard Taylor. Mechanical work in terrestrial locomotion: two basic mechanisms for minimizing energy expenditure. *American Journal of Physiology-Regulatory, Integrative and Comparative Physiology*, 233(5):R243–R261, 1977.
- [2] Ben Andrews, Bruce Miller, John Schmitt, and Jonathan E Clark. Running over unknown rough terrain with a one-legged planar robot. *Bioinspiration & biomimetics*, 6(2):026009, 2011.
- [3] B. Dadashzadeh, H. R. Vejdani, and J. Hurst. From template to anchor: A novel control strategy for spring-mass running of bipedal robots. In *Intelligent Robots and Systems (IROS 2014), 2014 IEEE/RSJ International Conference on*, pages 2566–2571, September 2014.
- [4] Pat Terry, Giulia Piovan, and Katie Byl. Reachability based and high order partial feedback linearization enforced control strategies for realistic series-elastic actuated hopping robots. *IEEE Transactions on Robotics*, 2016.
- [5] John Schmitt and Philip Holmes. Mechanical models for insect locomotion: dynamics and stability in the horizontal plane i. theory. *Biological cybernetics*, 83(6):501–515, 2000.
- [6] Daniel I. Goldman, Tao S. Chen, Daniel M. Dudek, and Robert J. Full. Dynamics of rapid vertical climbing in cockroaches reveals a template. *Journal of Experimental Biology*, 209(15):2990–3000, 2006.
- [7] John Schmitt, Mariano Garcia, RC Razo, Philip Holmes, and Robert J Full. Dynamics and stability of legged locomotion in the horizontal plane: a test case using insects. *Biological cybernetics*, 86(5):343– 353, 2002.
- [8] J Schmitt and S Bonnono. Dynamics and stability of lateral plane locomotion on inclines. *Journal of Theoretical Biology*, 261(4):598–609, 2009.
- [9] Jason M Brown, Bruce D Miller, and Jonathan E Clark. Classification of dynamical vertical climbing gaits. In Intelligent Robots and Systems (IROS), 2016 IEEE/RSJ International Conference on, pages 4816–4822. IEEE, 2016.
- [10] Goran A. Lynch, Jonathan E. Clark, Pei-Chun Lin, and Daniel E. Koditschek. A bioinspired dynamical vertical climbing robot. *The International Journal of Robotics Research*, 31:974–996, April 2012.
- [11] Marc H Raibert. Trotting, pacing and bounding by a quadruped robot. *Journal of biomechanics*, 23:7983–8198, 1990.
- [12] Sangbae Kim, Matthew Spenko, Salomon Trujillo, Barrett Heyneman, Daniel Santos, and Mark R

1		
2	Impa	act of Slope on Dynamics of Running and Climbing
3	ттра	ci of stope on Dynamics of Kunning and Climoting
4		
5		Cutkosky. Smooth vertical surface climbing with directional adhesion. <i>Robotics, IEEE Transactions</i>
6		on, 24(1):65–74, 2008.
7	[13] I	Hae-Won Park, Patrick M Wensing, and Sangbae Kim. High-speed bounding with the mit cheetah 2:
8		Control design and experiments. The International Journal of Robotics Research, 36(2):167-192, 2017.
9	[14] I	Bruce D Miller, Peter R Rivera, James D Dickson, and Jonathan E Clark. Running up a wall: the
10		role and challenges of dynamic climbing in enhancing multi-modal legged systems <i>Bioinspiration</i> &
11		Riomimotics 10(2):025005 2015
12	[15]	MH Deibert Legend Pohots that Palance Artificial Intelligence MIT Dross 1086
13	[15] 1	M.H. Kalden. Legged Robols indi Balance. Artificial Intelligence. Mil Press, 1980.
14	[16] 1	Hartmut Geyer, Andre Seyfarth, and Reinhard Blickhan. Compliant leg behaviour explains basic
15		dynamics of walking and running. Proceedings of the Royal Society of London B: Biological Sciences,
16		273(1603):2861–2867, 2006.
17	[17] N	MJ Spenko, G Clark Haynes, JA Saunders, Mark R Cutkosky, Alfred A Rizzi, Robert J Full, and Daniel E
18		Koditschek. Biologically inspired climbing with a hexapedal robot. Journal of Field Robotics, 25(4-
10		5):223–242, 2008.
20	[18] I	Bruce D Miller and Ionathan E Clark Towards highly-tuned mobility in multiple domains with a
20		dynamical lagged platform Rightspirgtion & biominatics 10(1):046001-2015
21	[10]	dynamical legged platform. <i>Bioinspiration & biointimetics</i> , 10(4).040001, 2013.
22	[19] 1	Max P Ausun, Jason M Brown, Charles A Young, and Jonathan E Clark. Leg design to enable dynamic
23		running and climbing on bobcat. In 2018 IEEE/RSJ International Conference on Intelligent Robots and
24		<i>Systems (IROS)</i> , pages 3799–3806. IEEE, 2018.
25	[20] A	Andrew A Biewener and Robert J Full. Force platform and kinematic analysis. In Biomechanics: a
26		practical approach, pages 45–73. January 1992.
2/	[21] I	Robert J Full and Michael S Tu. Mechanics of a rapid running insect: two-, four-and six-legged
28		locomotion. Journal of Experimental Biology, 156(1):215–231, 1991.
29	[22] [Robert I Full Reinhard Blickhan and I H Ting. Leg design in hexanedal runners. <i>Journal of Experimental</i>
30		Riology 158(1):360–300 1001
31	[22] 1	Biology, 138(1).509–590, 1991.
32	[23] 1	K Kram, B wong, and KJ Full. Infee-dimensional kinematics and limb kinetic energy of running
33		cockroaches. Journal of Experimental Biology, 200(13):1919–1929, 1997.
34	[24] \$	Shibendu Shekhar Roy and Dilip Kumar Pratihar. Study on energy consumption in turning motion of
35		hexapod walking robots. In Proceedings of the World Congress on Engineering, volume 3, pages 2349–
36		2354, 2011.
37	[25] J	Jonathan Clark, Daniel Goldman, Pei-Chun Lin, Goran Lynch, Tao Chen, Haldun Komsuoglu, Robert J
38		Full, and Daniel E Koditschek. Design of a bio-inspired dynamical vertical climbing robot. In <i>Robotics:</i>
39		Science and Systems, volume 1, 2007.
40	[26]]	Justin E Seinel Philin I Holmes and Robert I Full Dynamics and stability of insect locomotion: a
41	[20] 3	havenedel model for herizental plane motions. <i>Biological cybernatics</i> , 01(2):76, 00, 2004
42	[27]	A Wightermanuring and I Schwitt, Improving herizontal plane locamation via log angle control. Journal
43		A wickianiasuriya and J Schnintt. Inproving norizontal plane locomotion via leg angle control. <i>Journal</i>
44		of theoretical biology, 256(3):414–427, 2009.
45	[28] I	Katja D Mombaur, Richard W Longman, Hans Georg Bock, and Johannes P Schloder. Stable one-legged
46		hopping without feedback and with a point foot. In Proceedings 2002 IEEE International Conference
47		on Robotics and Automation (Cat. No. 02CH37292), volume 4, pages 3978–3983. IEEE, 2002.
48	[29] J	Jeffrey C Lagarias, James A Reeds, Margaret H Wright, and Paul E Wright. Convergence properties of the
49		nelder-mead simplex method in low dimensions. SIAM Journal on optimization, 9(1):112-147, 1998.
50	[30] [TM Kubow and RJ Full. The role of the mechanical system in control: a hypothesis of self-stabilization
51	[]	in hexanedal runners Philosophical Transactions of the Royal Society of London Series B. Biological
52	(Sciences 354(1385):840, 861, 1000
53	[21] [Sciences, 554(1565).647-601, 1777.
54	[31] 1	LH ring, Reinnard Blicknan, and Robert J Full. Dynamic and static stability in nexapedal runners. <i>Journal</i>
55		of Experimental Biology, 197(1):251–269, 1994.
56	[32]]	JE Clark and DE Koditschek. A spring assisted one degree of freedom climbing model. In Fast Motions
57		<i>Vin Biomechanics and Robotics</i> , pages 43–64. Springer Berlin Heidelberg, 2006.
58	[33] J	Jonathan E Clark and Mark R Cutkosky. The effect of leg specialization in a biomimetic hexapedal running
59		robot. Journal of dynamic systems, measurement, and control, 128(1):26-35, 2006.
60	T.	